#### **Lecture 13-14-15**

# MAIN HYDRODYNAMIC CALCULATIONS UNDER WATER DRIVE

Zone existence of water drive is above the saturation pressure it is characterized by constancy formation pressure, which is achieved due to penetrating of water in the oil reservoir or by pumping water from the surface. The theoretical basis for the calculation of development is discipline "Underground hydrogasmechanic".

The modern designing of oil and gas fields calls for simple calculations using licensed software and powerful computing facilities. However, simple models that we consider enable quality results quickly without long calculations based on more complex models.

#### **Schematization terms of development**

Oil deposits have in terms of irregular geometric shapes, and usually have a complex geological structure and the irregular configuration, the thickness of the reservoir is inconsistent in size, unequal is and properties of the reservoir (porosity, permeability, oil saturation) variety may be the properties of oil on different parts of the deposit. This complicates the calculations in the design development of oil fields. The development put the wrong geometry analytically cannot be calculated accurately.

For the approximate solving of this problem possible approximation of the actual deposit shapes such forms or parts of forms that are subjected to analytic calculation. In the calculations have to be subjected to the schematic design conditions, use model layer and liquid filtration processes in porous media.

#### Schematization is performed in four stages:

- schematization form deposit;
- schematization circuit WOC;
- - built spatial problem to the plane. Formation and bottom-hole pressures seams in the wells reduced to one plane (usually to the initial level WOC);
- - parameters of layer and properties of formation fluids.

Different circuit models of the reservoir, which are chosen depending on the degree of scrutiny and put the design phase, homogeneous layer, uniform area-layer, heterogeneous continuum of level-permeability and thickness of the layer of natural or probable distribution of these parameters.

#### **Schematization forms deposits**

In practice, there are examples of different forms: stripes, circles, ellipses shaped and their various combinations.

Any form of lay mainly in the calculations can be made to a strip or circle, or to a combination of these forms.

Schematization forms of oil are to replace the complex configuration put on deposit in the form of strips, rings, circle or a set of simple geometric shapes.

If the deposit for which hydrodynamic calculations performed is oval in shape, with the ratio of the axes A:B > 3 (where A – length of deposit, B - width of deposit), then the deposit is schematized by strip (Fig. 1).

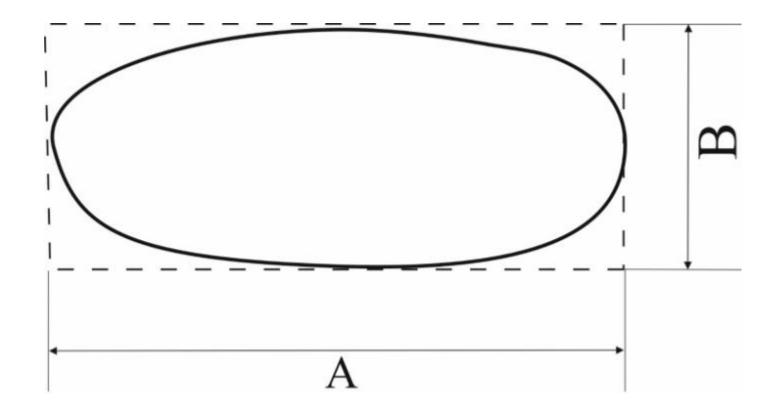


Figure 1 - Schematization forms deposits (strip deposit)

In order to calculated indicators met the real, in terms of schematic design must adhere to the following conditions:

- 1) reserves oil of real and schematic deposits should be the same;
- 2) perimeters of real and schematic deposits should be the same;
- 3) a number of wells schematic and real deposits must be the same.

Oval deposit, in which A:B < 3, schematized is ring or circle (Fig 2).

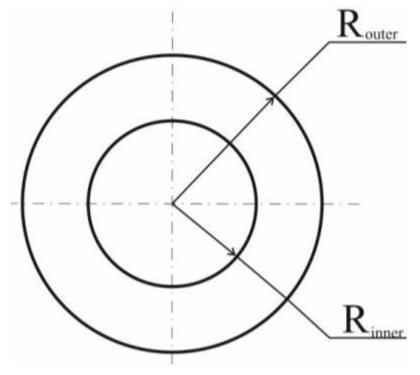


Figure 2 - Schematization forms deposits (circle deposit)

After completing schematic terms, we write

$$R_{outer} = \frac{P}{2\pi} \tag{1}$$

where P - the real perimeter of the oil deposit.

$$R_{inner} = \sqrt{R_{outer}^2 - \frac{F}{\pi}} \tag{2}$$

where F – the real area of the oil deposit.

If the real reservoir has a complex configuration, such deposit is schematized several elements square regular geometric shapes (Fig. 3)

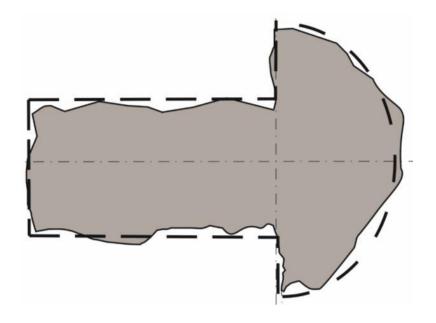


Figure 3 – Schematization complex forms deposits

This deposit is schematized in a strip and a semicircle. Calculations are made to strip and circle.

### Method of electrohydrodynamic analogy (EHDA) for solving hydrodynamic tasks

Simulation of hydrodynamic processes in the reservoir electric model is based on the analogy between hydrodynamic and electrical processes that have similar equations and describing both systems.

We write the equation of fluid inflow

$$Q = k_0 \cdot \Delta P = \frac{\Delta P}{\frac{1}{k_0}} \tag{3}$$

where  $k_0$  - productivity coefficient

Write Ohm's law

$$I = \frac{\Delta U}{R} \tag{4}$$

Making an analogy between the equations (3) and (4) we see that the analogy between hydrodynamic and electrical parameters is shown in the following:

 $\Box$  change in voltage between nodes electricity grid  $\Delta U$  similar to the pressure, that is

$$\Delta U \equiv \Delta P$$

□ current, which flows between the nodes of the grid is proportional to the amount of fluid that flows between the regions of the reservoir, that is

□ Finally, by analogy

$$R\equiv 1/k_0$$

This value is called the filtration resistance

The principle (EHDA) is based on generalization electric current laws and fluid flow in porous media. The principle is based on the community mathematical description of the processes occurring in the reservoir during the filtration process of fluids flow of electric current.

Based on the method (EHDA) professor Y. Borisov has developed a method of assembling the hydrodynamic equations for calculation processes of oil fields in the certain placement of wells. Using this method, you can easily make a determination equation flowrates and bottom-hole pressures for any of the options strip or circular rows of wells.

## Hydrodynamic calculations flowrates and bottom-hole pressures, and also validities of water-oil displacement without phase permeabilities

The task is solved for two cases

- when specified bottom-hole pressures and need to find flowrates
- when specified flowrates, and the need to find bottomhole pressures

Chance is another task statement when some asked find bottom-hole pressures and flow rates in other rows of wells

**Example 1.** We have strip deposit, which is being developed by three rows of wells. Define flowrates rows of wells, if known: bottom-hole pressures in the rows of wells, reservoir parameters and reservoir fluids and a number of wells (Fig.4).

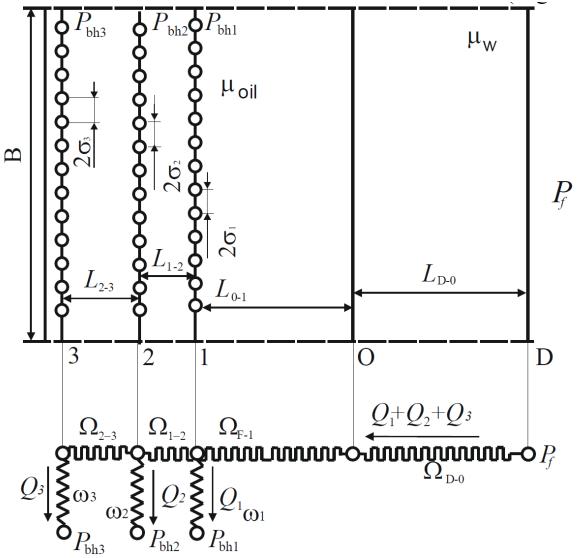


Figure 4 – Gydrodynamic and electrical schemes of strip deposit which is developeted by three rows of wells

Inflow to all wells can be seen as a parallel connection of conductors with identical resistances  $(\Omega + \omega)$ . Sophisticated filtering right seams between injection rows and production wells are presented using simple filtration flows.

Thus, the filtration inflow into the well can be submitted electrical equivalent circuit for calculating the resistance and use Ohm's and Kirchhoff's laws. Drawing calculation equation system development processes based on the second Kirchhoff's law.

We write equations (we have **3 equations** - number of equations in the system equals the number of rows of wells)

$$\begin{cases}
P_{f} - P_{bh1} = (Q_{1} + Q_{2} + Q_{3})\Omega_{D-0} + (Q_{1} + Q_{2} + Q_{3})\Omega_{0-1} + Q_{1}\omega_{1} \\
P_{f} - P_{bh2} = (Q_{1} + Q_{2} + Q_{3})\Omega_{D-0} + (Q_{1} + Q_{2} + Q_{3})\Omega_{0-1} + (Q_{2} + Q_{3})\Omega_{1-2} + Q_{2}\omega_{2} \\
P_{f} - P_{bh3} = (Q_{1} + Q_{2} + Q_{3})\Omega_{k-0} + (Q_{1} + Q_{2} + Q_{3})\Omega_{0-1} + (Q_{2} + Q_{3})\Omega_{1-2} + Q_{3}\Omega_{2-3} + Q_{3}\omega_{3},
\end{cases} (5)$$

where  $P_f$  – pressure drainage boundary (formation pressure), Pa;  $P_{bh1}$ ,  $P_{bh2}$ ,  $P_{bh3}$  - bottom-hole pressures the first, second and third rows of wells, Pa;  $Q_1$ ,  $Q_2$ ,  $Q_3$  – flowrates the first, second and third rows of wells, m³/s;  $\Omega_{D-o}$ ,  $\Omega_{o-1}$ ,  $\Omega_{1-2}$ ,  $\Omega_{2-3}$  - external filtration resistances, Pa·s/m³;  $\omega_I$ ,  $\omega_2$ ,  $\omega_3$  – internal filtration resistances, Pa·s/m³.

**External filtration resistances** for the strip deposit are defined by formulas

$$\Omega_{D-0} = \frac{\mu_w l_{k-0}}{Bkh};\tag{6}$$

$$\Omega_{1-2} = \frac{\mu_{oil} l_{1-2}}{Bkh}; (7)$$

$$\Omega_{0-1} = \frac{\mu_{oil} l_{0-1}}{Bkh}; \tag{8}$$

$$\Omega_{2-3} = \frac{\mu_{oil} l_{2-3}}{Bkh};\tag{9}$$

where  $\mu_w$ ,  $\mu_{oil}$  – coefficients of dynamic viscosity of water and oil, respectively, Pa·s, for water  $\mu_w$ =1 mPa·s;  $l_{D-0}$ ,  $l_{0-1}$ ,  $l_{1-2}$ ,  $l_{2-3}$  – distances, m; B – width of deposit, m; k – coefficient of permeability, m<sup>2</sup>; h – layer thickness, m.

**Internal filtration resistances** not depend on the form of deposit and defined by formulas

$$\omega_{l} = \frac{\mu_{oil}}{2\pi k h n_{l}} \ln \frac{\sigma_{l}}{\pi r_{rwl}}; \tag{10}$$

$$\omega_2 = \frac{\mu_{oil}}{2\pi k h n_2} \ln \frac{\sigma_2}{\pi r_{rw2}}; \tag{11}$$

$$\omega_3 = \frac{\mu_{oil}}{2\pi k h n_3} \ln \frac{\sigma_3}{\pi r_{rw3}}; \qquad (12)$$

where  $\sigma_1$ ,  $\sigma_2$ ,  $\sigma_3$  - the half distance between the wells for each row, m;  $n_1$ ,  $n_2$ ,  $n_3$  - amount of wells in each row.

Half distance between the wells is defined by the formula

$$\sigma_i = \frac{B}{2n_i} \tag{13}$$

Solving equation (5) determine the flowrates of rows of wells The system of equations (5) can make another method

$$\begin{cases}
P_f - P_{bh1} = (Q_1 + Q_2 + Q_3)\Omega_{D-0} + (Q_1 + Q_2 + Q_3)\Omega_{0-1} + Q_1\omega_1 \\
P_{bh1} - P_{bh2} = (Q_2 + Q_3)\Omega_{1-2} + Q_2\omega_2 - Q_1\omega_1 \\
P_{bh2} - P_{bh3} = Q_3\Omega_{2-3} + Q_3\omega_3 - Q_2\omega_2.
\end{cases} (14)$$

**Example 2.** We have circle deposit, which is being developed by two rows of wells. Define bottom-hole pressures in the rows of wells, if known: flowrates rows of wells, reservoir parameters and reservoir fluids and the number of wells (Fig. 5).

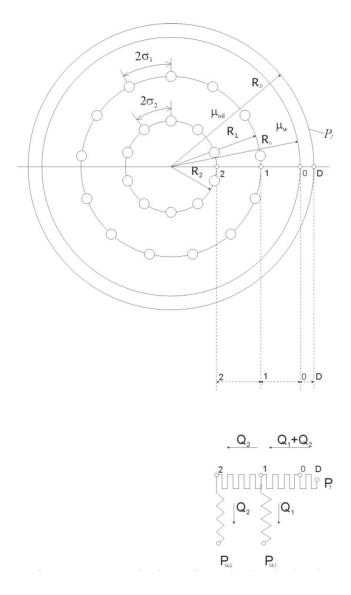


Figure 5 - Gydrodynamic and electrical schemes of circle deposit which is developeted by two rows of wells

Using the principle EGDA we write a system of equations

$$\begin{cases} P_f - P_{bh1} = (Q_1 + Q_2)\Omega_{D-0} + (Q_1 + Q_2)\Omega_{0-1} + Q_1\omega_1; \\ P_f - P_{bh2} = (Q_1 + Q_2)\Omega_{D-0} + (Q_1 + Q_2)\Omega_{0-1} + Q_2\Omega_{1-2} + Q_2\omega_2. \end{cases}$$
(15)

External filtration resistances for circle deposit are defined by formulas

$$\Omega_{D-0} = \frac{\mu_w \ln \frac{R_D}{R_0}}{2\pi kh}; \tag{16}$$

$$\Omega_{0-1} = \frac{\mu_{oil} \ln \frac{R_0}{R_1}}{2\pi k h};$$
(17)

$$\Omega_{1-2} = \frac{\mu_{oil} \ln \frac{R_1}{R_2}}{2\pi k h}.$$
 (18)

where  $R_D$  - radius drainage boundary, m;

 $R_{\theta}$  - radius WOC, m;

 $R_1$ ,  $R_2$  - radiuses first and second rows of wells, m.

Half distance between the wells for circle deposit is defined by the formula

$$\sigma_i = \frac{\pi R_i}{n_i} \,, \tag{19}$$

where  $n_i$  - the number of wells in a row.

Solving equation (15) determines the bottom-hole pressures of rows of wells.

From system (5) determining the flowrate of each row of wells can determine the total flow rate of deposit

$$Q_{sum} = \sum_{i=1}^{n} Q_i \tag{20}$$

and flowrate in a row, assuming they have the same flowrates

$$q_i = Q_i/n_i. (21)$$

If the change number rows of production wells, the system of equation (5) differing only in the number of equations. The structure and order of the equations remain the same. Directly from the equation (5) a connection between Q and n (Fig. 6).

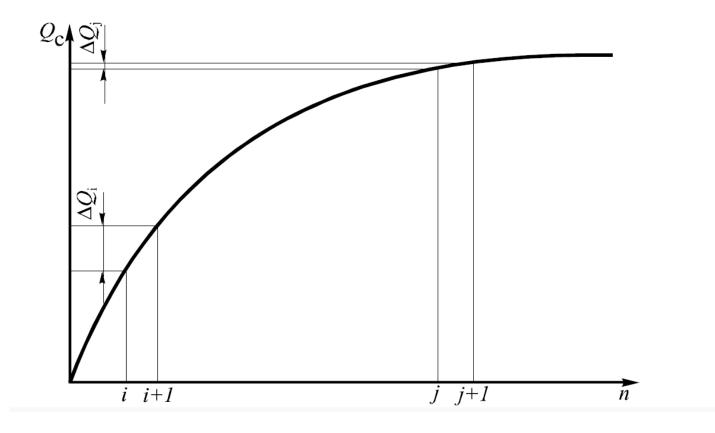


Figure 6 - The dependence of oil production from the number of wells

The relationship between the number of wells and the summary liquid production is nonlinear. When the number of wells may come a time when increasing the number of wells does not increase production. Then for increasing oil production need to change the operating conditions of wells that change bottom-hole pressure.

Calculation time of development are based on data on the flow rates of wells. Full time of development consists of separate time of displacement calculation WOC from its initial position to the line of the first row of wells, from the line the first row of wells to the line of the second row of wells, and etc.

Time displacement of the oil-bearing path from one position to another is given by the formula

$$t = \frac{Q_{res.}}{\sum_{i=n}^{\infty} Q_i} \frac{1}{1 - \frac{n_w}{100}},$$
(22)

where  $Q_{res}$ . - recoverable oil reserves in the area of the substitution of oil with water, determined by the Zhdanov's formula;

 $\sum Q_i$  - the total flow rate of all wells operating at a given time;

 $n_w$  – water-cut, %.

Oil reserves in the zone of displacement for strip deposit

$$Q_{res.} = Bh \, lm \rho_0 \eta \,\,, \tag{23}$$

where l - the distance between the WOC, m;

 $\rho_{o}$ - initial oil-saturation;

 $\eta$  - oil recovery factor.

Oil reserves in the zone of displacement for circle deposit

$$Q_{res.} = \pi (R_1^2 - R_2^2) h m \rho_0 \eta , \qquad (24)$$

where  $R_1$ ,  $R_2$  - radiuses WOC in two positions at the beginning and end of the billing period, m.

Calculation time development of deposit is as follows. Determine the time moving of WOC from the initial position to the first row of wells

$$t_1 = \frac{Q_{res._1}}{\sum_{i=n_1} Q_i} \frac{1}{1 - \frac{n_{w_1}}{100}}.$$
 (25)

Then determine the time moving of WOC wells from the first row of the second

$$t_2 = \frac{Q_{res.2}}{\sum_{i=n_2} Q_2} \frac{1}{1 - \frac{n_{w2}}{100}}.$$
 (26)

The total time of development

$$T = \sum_{i=1}^{i=k} t_i \tag{27}$$

where k – amount rows of wells.