



*Ivano-Frankivsk National Technical
University of Oil and Gas*



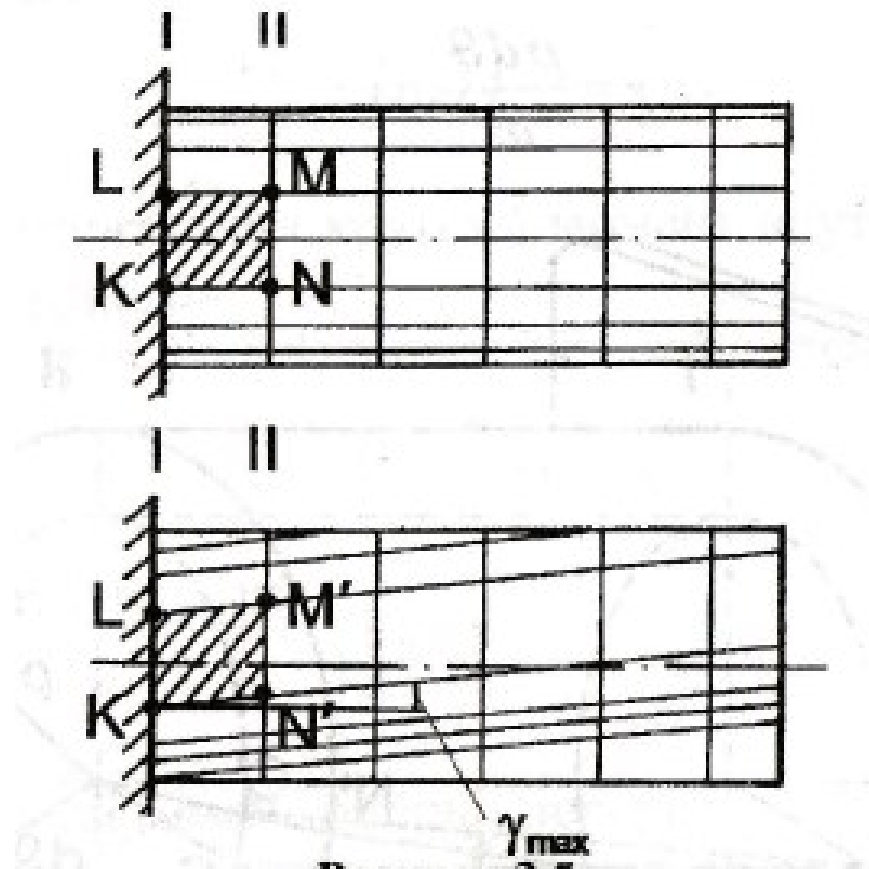
**Kielce University
of Technology**

Torsion

Dr. of Eng. Sci., prof. Ruslan Rachkevych

Torsion

What the stresses does appiar in a cross-section?



1) The rectangular grid will be transformed into a grid of parallelograms. This indicates the presence **of shear stresses in the cross-sections of the beam** .

According to the law of parity of shear stresses, the shear stresses also appear in longitudinal sections.

2) The distance between circles, for example, I and II, remains unchanged. The length of the beam and its diameter do not change also.

Torsion

What the stresses does appiar in a cross-section?

Torsional deformation is caused by pairs of forces whose planes of action **are perpendicular** to the axis of the rod.

These pair of forces are called as **torque**.

Rods that work by torsion are called as **shafts**.

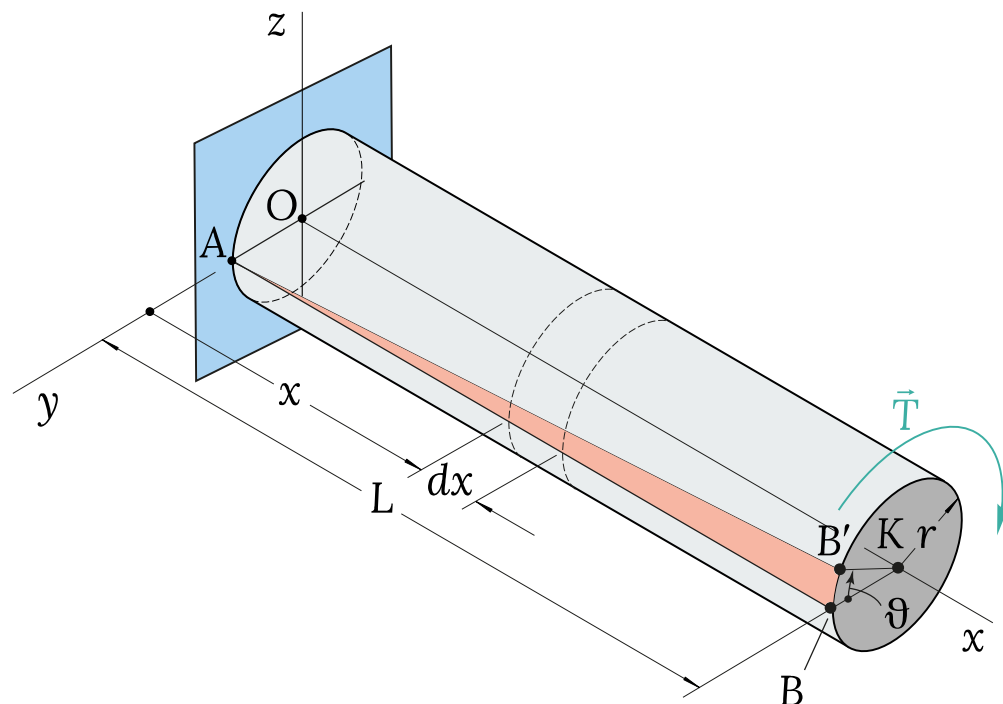
When bodies that are applied by the torsion are analysed, the hypothesis **of plane sections is accepted**:

If rod with circular cross-section is tersed, its cross-sections that were flat before deformation, remain flat after deformation and rotate one relative to anouther by some angle.

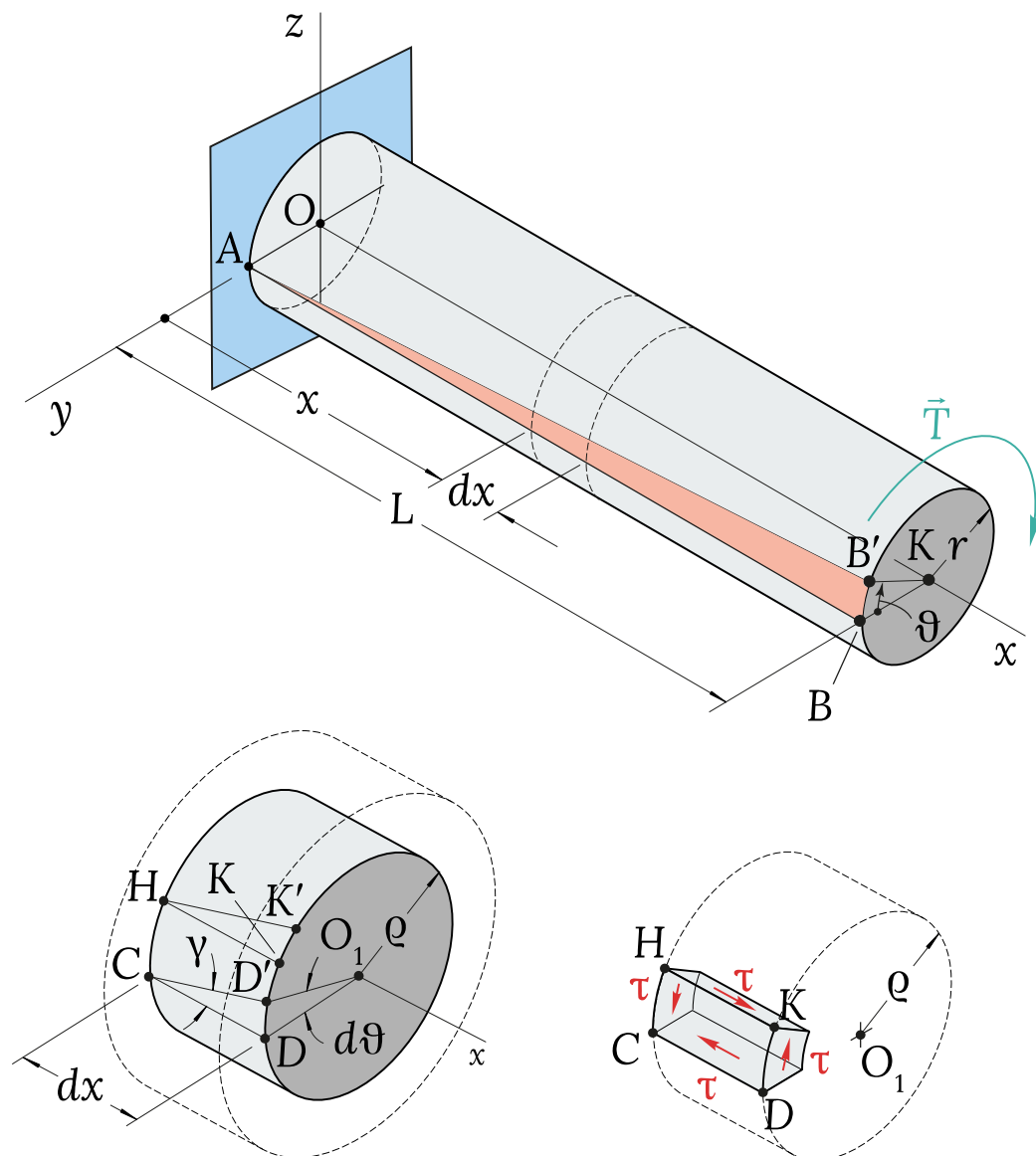
In general, when studying a rod for torsion, the following **assumptions are used**:

1. Circular cross-sections remain flat (do not deform) and perpendicular to the shaft axis.
2. Cross sections are not deformed (there is no deformation in the plane of the cross section).
3. The distances between the cross-sections do not change (axial normal deformation is zero).

Based on these assumptions, the following rule can be formed: each cross-section of the shaft rotates as a solid plane around the shaft axis.



Torsion



What the stresses do appear in a cross-section?

Consider a shaft of circular cross-section with radius r and length L , one end of which is rigidly fixed at point O , and a torque T is applied to the other. Under the action of torque T , the line AB drawn on the surface of the shaft parallel to its axis takes the form of a helical line AB' , characterized by a twist angle ϑ .

Consider a part of a shaft of extremely small length dx , located at a distance x from point O . Discarding the outer layer, we consider what deformations will occur on the cylindrical part, characterized by a radius ρ from the shaft axis. The magnitude of the displacement of point D , located at the edge of the element, is equal to:

$$DD' = \gamma dx = \rho d\vartheta,$$

From here:

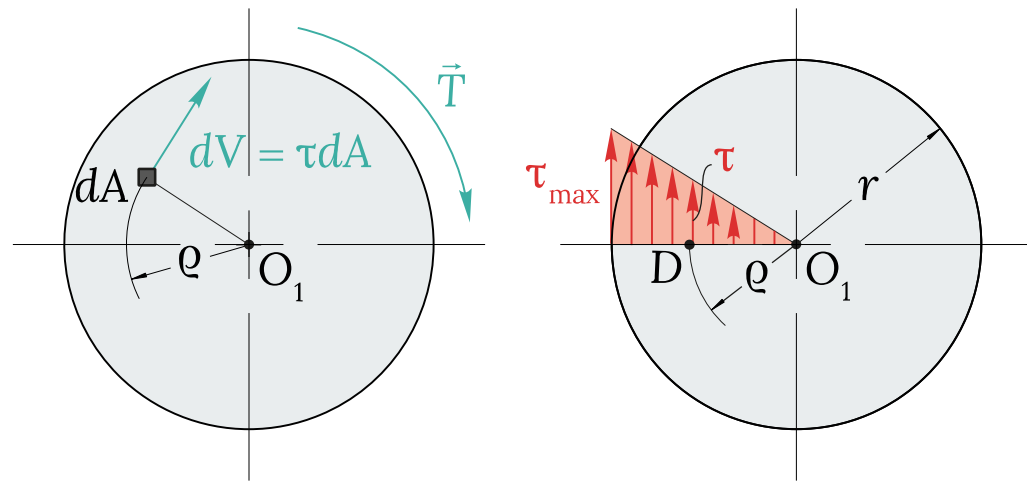
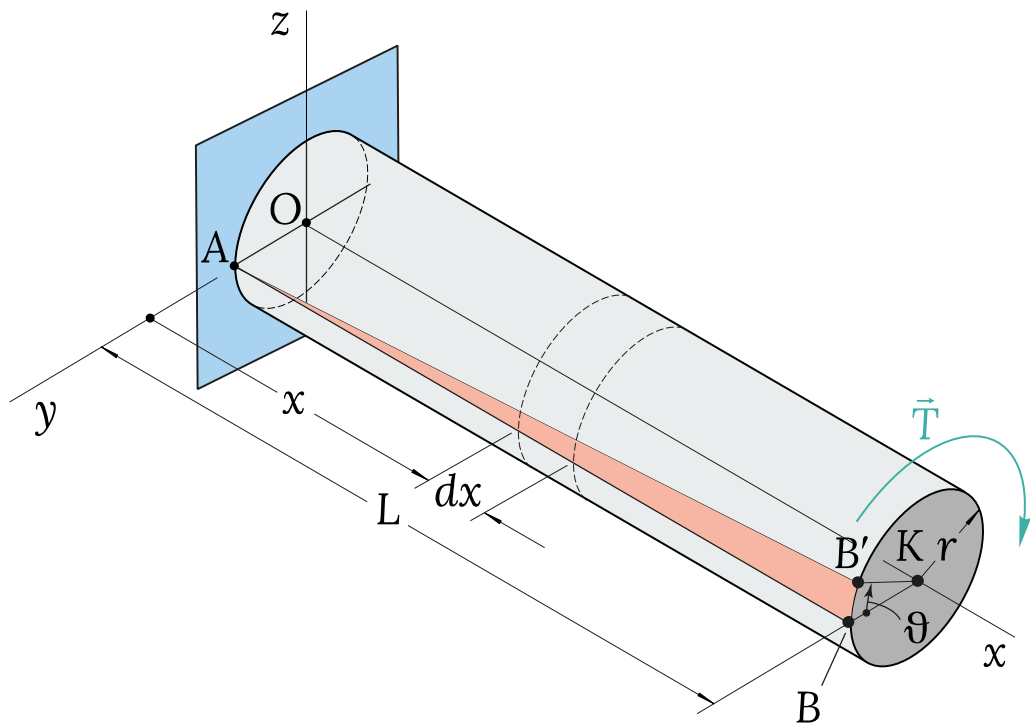
$$\gamma = \rho \frac{d\vartheta}{dx}.$$

Considering the deformation of the part $CDHK$ of the cylindrical surface, under the action of the torque in the shaft, only relative shear deformations arise, the magnitude of which is equal to τ . Applying Hooke's law, we obtain:

$$\tau = G\gamma \quad \tau = G \frac{d\vartheta}{dx} \rho,$$

where G is the shear modulus.

Torsion



What the stresses do appiar in a cross-section?

Let us consider an elementary area dA in the cross section of the shaft. Taking into account the above, a tangential stress τ will act in it. Therefore, the force that will arise in the area is equal to:

$$dV = \tau dA.$$

Based on the principle of symmetry, these forces will balance each other and create a moment of force in the plane whose magnitude is equal to:

$$T = \int_A \rho \tau dA = \int_A \rho^2 G \frac{d\theta}{dx} dA = G \frac{d\theta}{dx} \int_A \rho^2 dA = G \frac{d\theta}{dx} I,$$

As a result, we get:

$$\frac{T}{GI} = \frac{d\theta}{dx}.$$

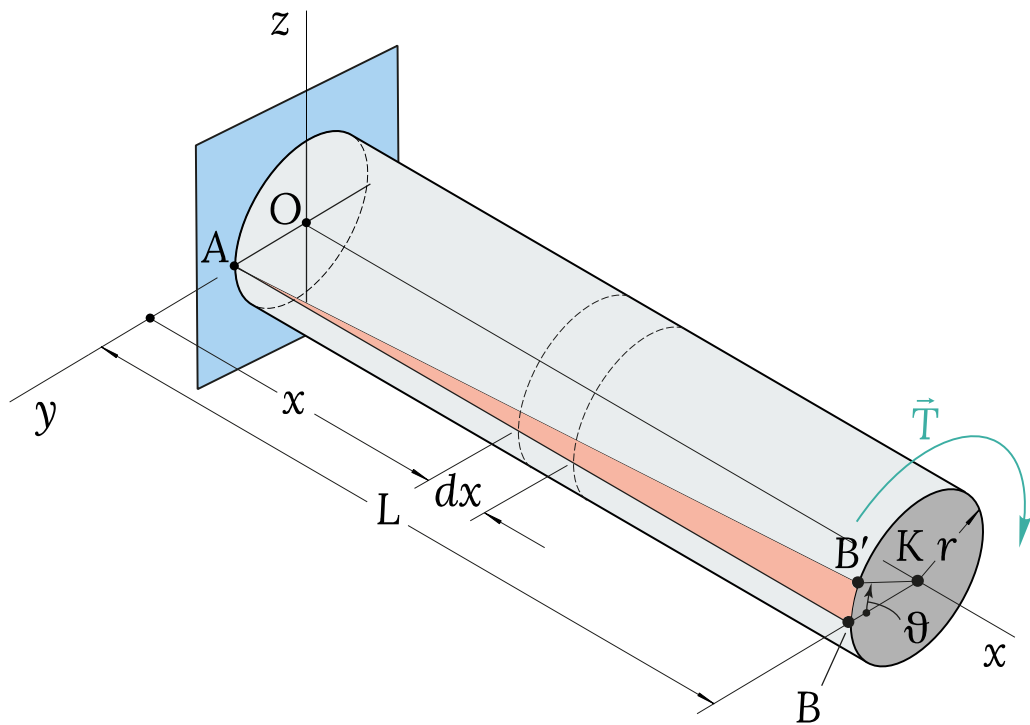
Therefore, the value of the twist angle is equal to:

$$\theta = \int_0^L \frac{T}{GI} dx,$$

Given the same cross-sectional area and torque acting on the section, we obtain:

$$\theta = \frac{TL}{GI}.$$

Torsion



Analysis of the stress distribution shows that their maximum values occur at the greatest distance relative to the axis, therefore:

$$\tau_{\max} = G \frac{d\vartheta}{dx} r.$$

From here:

$$\tau_{\max} = G \frac{d\vartheta}{dx} r = \frac{Tr}{I} = \frac{T}{W},$$

where W is the polar moment of resistance during torsion. For a shaft of circular cross-section it is equal to

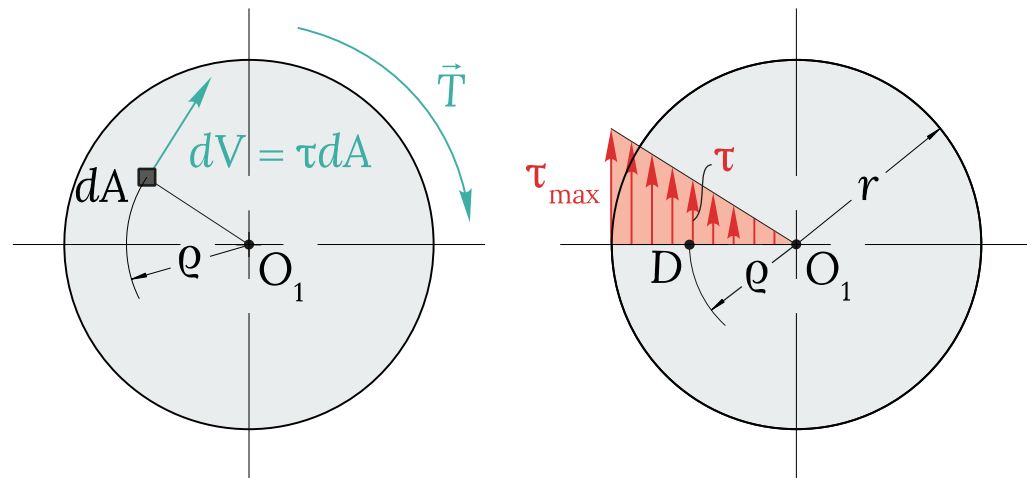
$$W = \frac{\pi d^3}{16}$$

The condition for the strength of the shaft during torsion is the requirement that the maximum tangential stresses in the rod must not exceed the permissible ones:

$$[\tau] \geq \tau_{\max} = \frac{T}{W}.$$

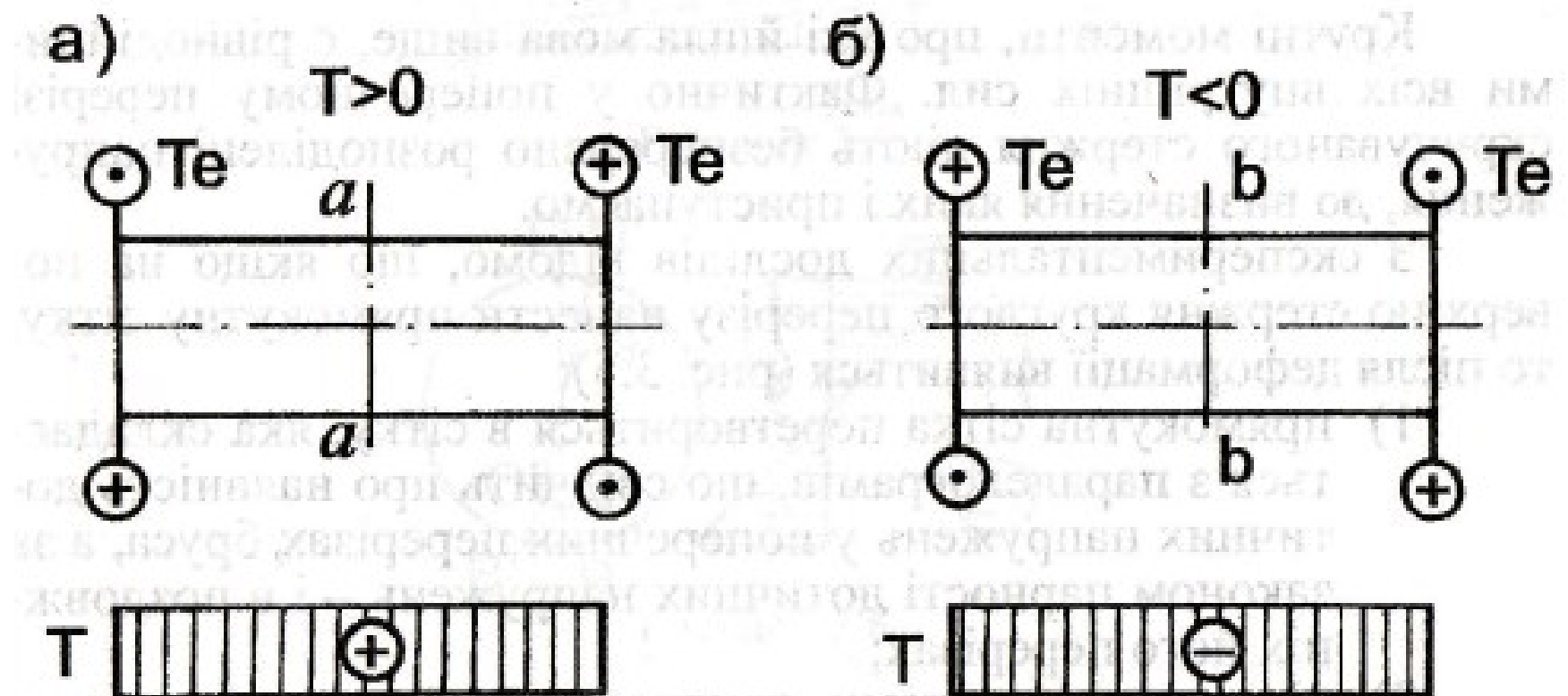
The condition for calculating the shaft for rigidity is the requirement that the relative twist angle should not be greater than the permissible

$$\left[\frac{d\vartheta}{dx} \right] \geq \vartheta_{\max} = \frac{T}{GI}.$$



Torsion

Torque diagram (rule of signs)



Torsion

Torque diagram

$$T_{AB} = -10 \text{ kNm};$$

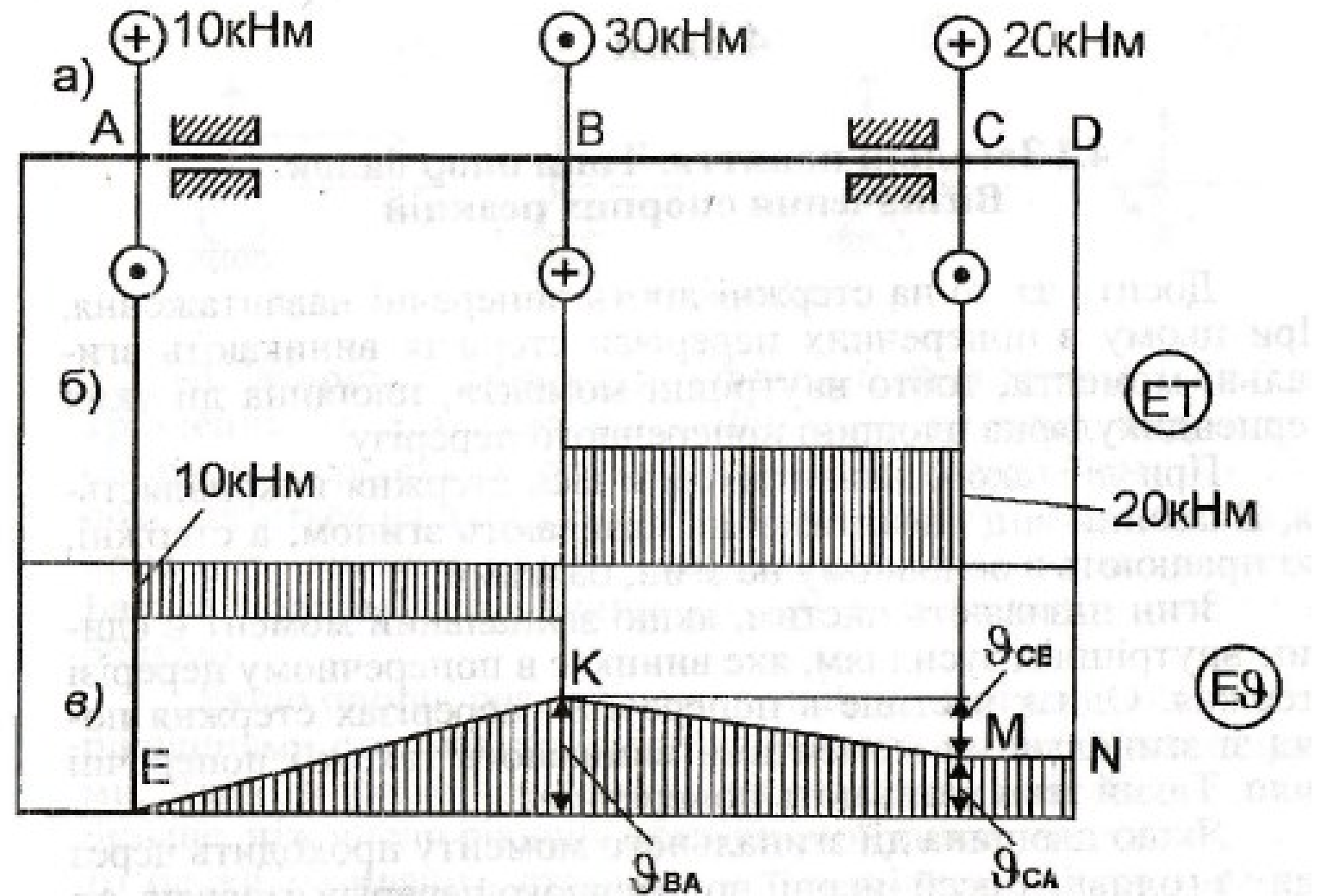
$$T_{BC} = -10 + 30 = 20 \text{ kNm};$$

$$T_{CD} = -10 + 30 - 20 = 0 \text{ kNm}.$$

$$\varphi_{BA} = T_{AB} l_{AB} / (G I_p);$$

$$\varphi_{CB} = -T_{BC} l_{BC} / (G I_p);$$

$$\varphi_{CA} = \varphi_{CB} + \varphi_{BA}.$$





Erasmus+

Thanks for your attention!

Email: ruslan.rachkevych@nung.edu.ua

15 Karpatska Street
Ivano-Frankivsk 76019
UKRAINE
Phone: +38 03422 4-22-64
E-mail: admin@nung.edu.ua
www.nung.edu.ua